

GLUON PROPAGATOR IN THE LANDAU GAUGE FIXED LATTICE QCD SIMULATION

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Abstract

We measured the gluon propagator in the Landau gauge fixed QCD Langevin simulation[2] and studied the infra-red behaviour of the gluon propagator. The $(4^3 \times 8)$ lattice simulation was done for quenched $\beta = 3, 4$ and 5 and unquenched $\beta = 4, \kappa = 0.1, 0.15$ and 0.2, using each 100 independent samples. The Landau gauge fixing was done by an extension of the Fourier acceleration method with the condition $|\text{div}A| < 10^{-4}$, and the field A is related to the link variable by $U = \exp A$ instead of the usual U-linear definition. We confirmed gauge fixing with smearing preconditioning[3] works perfectly for the purpose of finding the global minimum of the squared norm of the gauge field when β is large (e.g. $\beta = 5$). Our simulation results suggests the possibility of a realization of the infra-red behaviour of the Gribov-Zwanziger theory.

1 Introduction

In 1978 Gribov showed that the fixing of the divergence of the gauge field in non-Abelian gauge theory does not uniquely fix its gauge. The restriction of the gauge field such that its Faddeev-Popov determinant is positive (Gribov region) is necessary yet insufficient for excluding the ambiguity.

The restriction to the fundamental modular region, i.e. minimizing the norm of the gauge field, cancels the infrared singularity of the perturbation theory and makes the gluon to have the complex mass.

$$\begin{aligned} D_{\mu\nu}(k) &= \frac{1}{n} \sum_{x=\mathbf{x},t} e^{-ikx} \text{Tr} \langle A_\mu(x) A_\nu(0) \rangle \\ &= G(k^2) (\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) \end{aligned} \quad (1)$$

$$G(k^2) = \frac{k^2}{k^4 + \kappa^4} = \frac{1}{2} \left(\frac{1}{k^2 + i\kappa^2} + \frac{1}{k^2 - i\kappa^2} \right) \quad (2)$$

where $n = N^2 - 1$ is the dimension of the adjoint representation of the $SU(N)$ group.

In order to measure the propagator on a lattice, it is necessary to avoid the Gribov copy. We confirmed that it is possible by the smeared gauge fixing[3], when $\beta = 5$ in the case of $4^3 \times 8$.

2 The gluon propagator

On the lattice we define an $SU(N)$ lattice action for $S = S_{gauge} + S_{fermi}$ in the pseudo-fermion scheme. We measure

$$\begin{aligned} D_T(\mathbf{k}, t) &= \frac{1}{2n} \sum_{j=1}^2 \sum_T \text{Tr} \langle \tilde{A}_j(\mathbf{k}, T+t) \tilde{A}_j(-\mathbf{k}, T) \rangle \\ &= \frac{1}{2n} \sum_{j=1}^2 \sum_T \sum_{\mathbf{x}} e^{-i\mathbf{k}\mathbf{x}} \text{Tr} \langle A_j(\mathbf{x}, T+t) A_j(\mathbf{0}, T) \rangle \end{aligned} \quad (3)$$

where \tilde{A}_j is the Fourier component of A_j , and j is summed over transverse polarization with respect to \mathbf{k} .

The gauge field on links is defined as $e^{A_{x,\mu}} = U_{x,\mu}$, where $A_{x,\mu}^\dagger = -A_{x,\mu}$. The gauge transformation is $e^{A_{x,\mu}^g} = g_x^\dagger e^{A_{x,\mu}} g_{x+\mu}$, $\phi_x^g = g_x^\dagger \phi_x$ where $g_x = e^{\epsilon_x}$ and ϵ is a traceless antihermitian

The connected part of the propagator is defined by the normalized Fourier transform

$$\tilde{A}_\mu^a(\mathbf{k}, T) = \frac{1}{\sqrt{N_x^3}} \sum_x A_\mu^a(\mathbf{x}, T) e^{-i\mathbf{k} \cdot \mathbf{x}} \quad (4)$$

as

$$\begin{aligned} G_{\mu\nu}(\mathbf{k}, t)_c &= \sum_{T,a} \langle \tilde{A}_\mu^a(\mathbf{k}, T+t) \tilde{A}_\nu^a(\mathbf{k}, T) \rangle^* \\ &- \sum_{T,a} \langle \tilde{A}_\mu^a(\mathbf{k}, T+t) \rangle \langle \tilde{A}_\nu^a(\mathbf{k}, T) \rangle^* \end{aligned} \quad (5)$$

where $\langle \tilde{A}_\mu^a(\mathbf{k}, T) \rangle$ specifies the expectation value of the gauge field in the Landau gauge. For this evaluation it is necessary to fix the global gauge transformation.

We fix the global gauge such that the average over T of the zero-mode of a sample: $\frac{1}{3N_T} \sum_{T,i} \tilde{A}_i^m(\mathbf{0}, T) \lambda_m$, where λ is the SU(3) Gell-Mann matrix, is diagonalized and the smallest eigenvalue appears at the top and the largest eigenvalue at the bottom. We observed that the 2nd term of $G_{ii}(\mathbf{0}, t)_c$ is almost independent of t and that the value is several % of the 1st term of $G_{ii}(\mathbf{0}, 0)_c$. The value is larger for the unquenched case as compared to the quenched case. The Fourier transform of $D_T(\mathbf{0}, t)$ in the unquenched $4^3 \times 8$ and in the quenched $8^3 \times 16$ lattice simulation [5] suggest the suppression of the infrared part of the gluon propagator.

In the Stingl's factorizing-denominator rational approximants (FDRA)

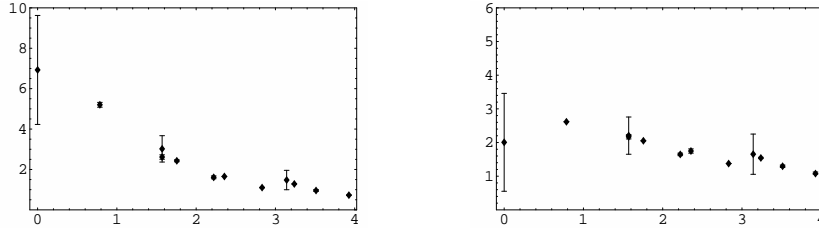


Figure 1: The gluon propagator $D_T(p)$. Ordinates are the euclidean 4-momentum p in the unit of a^{-1} . $\beta = 4, \kappa = 0$ (left) and $\kappa = 0.2$ (right). $4^3 \times 8$ lattice.

method[4], the transverse gluon propagator is expressed in the 2nd order as

$$D_T(p^2)^{[r=3]} = \frac{\rho}{p^2 + u_+ \Lambda^2} + \frac{\tau}{p^2 + v_+ \Lambda^2} + c.c. \quad (6)$$

We compared the lattice fourier transform of the above expression with $D_T(\mathbf{k}, t)$ and observed that in the confinement region there appears a pair of complex zero near $p^2 = 0$ as is suggested by the eq (2).

3 Discussion and outlook

In the Landau gauge, the transverse gluon propagator in the infrared region is finite at $p^2 = 0$ [6, 7] and the position of the zero of $D_T(p^2)$ approaches $p^2 = 0$ as the lattice size increases, while the propagator of the Faddeev-Popov ghost which is measured in the quenched $8^3 \times 16$ lattice shows the infrared singularity. These results support the possibility of a realization of the confinement mechanism of the Gribov-Zwanziger theory.

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